and were the discovery (now fifty years old) of the test-bearing protrochal stage of Dentalium, lately observed by Drew to be passed through by Yoldia and by Pruvot by a Dondersia, but recognised, we should not find the Chitons referred to a subclass of the Gastropoda and the "Solenogastres" accorded a class distinction. To this developmental stage, the discovery of which has dealt the death-blow to the idea of a Rhipidoglossan affinity of the Pelecypoda, and which, we trow, will ere long be extended to other groups, our authors should have directed attention. Had they done so, but three lines would not have sufficed for the Scaphopoda, and Spirula would not have been dismissed as a mere name.

Turning to palæontology, the non-recognition of the recent discovery in the Trilobites of nauplius characters deprives the authors' treatment of this larva of all force. And, similarly, had the Eurypterid forms recently described by Holm from the Russian Silurian, by Beecher from the Cambrian, and the Scorpionid genus Palæophonus, met with recognition, Limulus could not in justice have been once more relegated to the Arachnida. The absence in the present book of all mention of the Odontorinthes and Archæopteryx, of the Anomodontia, the Plesiosauria, and other leading fossil forms which might be named, is a serious omission, but even this does not excuse the non-reference to so important a group as the living Sphargidæ. Embryology and palæontology are branches of morphology coequal with the rest, and, so far as they reveal facts of primary significance, they should be dealt with as elementary subjects. Lack of appreciation of this principle is the weakest feature of the present work, which is, curiously enough, written with a special view to the requirements of the American student, who, of all beginners, is brought up in a palæontological air, and for whose benefit examples, wherever possible, are drawn from American as well as British animals.

Allowing for this serious defect, the book can be confidently recommended as well written and trustworthy, so far as it goes. It has been compiled at great pains, and its style leaves little to be desired. We wish it success and a speedy passage into a second edition; and, in anticipation of this, we would recommend to the authors' consideration the need of revision of such definitions as that of the endoderm cell (p. 48) as "tall"; of the blood-vessels (p. 89) as "chinks"; the replacement of the term "rudiment" on p. 259 by blastema; and certain other loosenesses which are self-evident. It is pertinent to this to remark that in some of their recent attempts at revised terminology, the zoologists of the Cambridge school have been none too successful. Thus, we note in the account of the life-history of the New Zealand reptile Sphenodon, given in the recently published natural history volume on "Amphibia and Reptiles," that the writer has substituted the word "æstivation" for what its discoverer rightly termed a hibernation. Is it possible that he has temporarily confused the southern summer with our own?

Of the illustrations, it may be said that figs. 266, 289 and 299 are examples which are poor, and might well be replaced; the statement that of the 32,000 "known species of Vertebrata" some 10,000 are Teleostei is surely excessive.

## *MATHEMATICAL TEXT-BOOKS IN THE UNITED STATES.*

*College Algebra.* By J. H. Boyd, Ph.D. Pp. xxii+788. (Chicago: Scott, Foresman and Co., 1901.)

WE cannot obtain a complete view of the state of mathematical studies in a country merely by examining the text-books and treatises which are in vogue there; but we do, in this way, gain a good deal of information about the aims and standards of its mathematical teachers. Dr. Boyd's treatise illustrates very well the qualities and defects of American methods, and suggests a few general remarks, as well as particular criticisms, which may not be out of place.

First of all, it must be acknowledged that the excellences of the better class of mathematical authors in the United States greatly outweigh their deficiencies. The American student is alert and inquisitive; he is neither impervious to new ideas, nor unwilling to make experiments. Moreover, teachers and students alike regard mathematics in the proper spirit—as a science which has, indeed, a venerable history, but is at the same time living and progressive, with ever new developments and ever fresh applications to the needs of man. Many, if not most, of the leading mathematicians in the States have studied in Germany, and have thus become acquainted with the work of Kronecker and Weierstrass and the far-reaching influence of this upon function-theory and the foundations of analysis. In elementary geometry, too, they are not the slaves of tradition, as we are; and it is not impossible that they may ultimately give us the ideal class-book in geometry for which we are waiting.

Dr. Boyd, in his preface, accepts the modern standard of rigour, and in his choice of topics combines the indispensable rudiments with those developments and applications which are really important. The general scope of his book may be indicated by saying that Book I. deals with the fundamental laws of operation; II. with equations of the first degree; III. with indices, surds and complex quantities; IV. with quadratic equations; V. with proportion, progressions and logarithms; VI. with induction, permutations and combinations, and the binomial expansion for a positive integral exponent; VII. with limits and series; VIII. with the properties of determinants and the elementary theory of equations.

After proving the fundamental laws of operation for the cases where they are arithmetically intelligible, the author extends them by purely formal definitions; thus $(a-b)$ is defined by the formal equivalence $(a-b)+b=a$. This is unobjectionable, but seems to us to require more justification than Dr. Boyd explicitly gives. He appeals to the "principle of permanence of form," but this "principle" remains practically an assumption. No doubt it would be extremely tedious to give (what we think has never been done) a complete logical proof that the application of the generalised laws of operation never involves an inconsistency; still, something more might have been done to help the reader to apprehend the reasonableness of the assumption.

Again, Dr. Boyd is not always consistent with himself. Thus, in the chapter on fractions, he begins with the formal definition $\frac{a}{b}\times b=a$; he subsequently says that

4/7 means that a group of 7 things is regarded as a unit group out of which 4 things are taken; and finally gives a proof of the equivalence of 4/7 and 12/21 by means of a graduated scale. This is mixing up three different ways of looking at the matter in a fashion which is very likely to cause confusion. And, so far as his "group" definition goes, he gives it in an imperfect form which is not immediately applicable to improper fractions and which fails to account for the equivalence of a pair such as 4/7 and 12/21.

Another chapter to which we naturally turn is that on irrational numbers and limits. Irrational numbers are treated, after Cantor, as the limits of sequences; and the discussion is satisfactory so far as it goes, though it might well be made rather more complete and is occasionally rather illogical. Thus, for instance, in the early part of the chapter it is said that the ordinary rule for finding a square root, when applied to 2, leads to the inequalities

$$1<\sqrt{2}<2, \quad 1\cdot4<\sqrt{2}<1\cdot5, \quad 1\cdot41<\sqrt{2}<1\cdot42,$$

and so on. As thus stated, the proposition is a pure *petitio principii*. The sequence $(1, 1\cdot4, 1\cdot41, \ldots)$ is convergent, and may be rationally combined with other such sequences according to Cantor's rules; therefore it may be regarded as a number. By definition

$$(1, 1\cdot4, 1\cdot41, \ldots)^2=(1^2, 1\cdot4^2, 1\cdot41^2, \ldots),$$

and this sequence can be proved to be equivalent to 2; therefore $\sqrt{2}$ is an appropriate symbol for $(1, 1\cdot4, 1\cdot41, \ldots)$. We must not begin by assuming the existence of $\sqrt{2}$ as an arithmetical quantity. The proof that sequences obey the laws of operation is put very briefly, and when we turn to the chapter on surds, we find that such an equivalence as $\sqrt{2}\,.\,\sqrt{3}=\sqrt{6}$ is justified, not by the use of sequences, but by a reference to the purely formal law of indices. Here, again, we have a rather unfortunate association of two entirely different notions. If, for any purpose, we like to introduce a symbol $\theta$ such that $\theta^2=2$, every rational function of $\theta$ can be reduced, by formal processes, to the shape $P+Q\theta$, where P and Q are independent of $\theta$; this is quite independent of the question whether $\theta$ can be properly regarded as a number or not; still less does it assign to $\theta$ its place in the arithmetical continuum.

Dr. Boyd's chapter on the binomial theorem for any exponent deserves attention, because, although it requires supplementing, it is novel, at least in a text-book, and may prove to be a good way of explaining the theorem to the college student. Let $p/q$ be a positive rational fraction; then

$$(1+x)^{p/q}=\sqrt[q]{(1+px+\tfrac{1}{2}p(p-1)x^2+\ldots+x^p)}.$$

Now it can be shown, as Dr. Boyd indicates without going into detail, that we can, by a process which is, in fact, Horner's method, determine a polynomial

$$y=1+\frac{p}{q}x+c_2x^2+c_3x^3+\ldots+c_mx^m,$$

such that

$$(1+x)^p-y^q=R=Ax^{m+1}+Bx^{m+2}+\ldots+Lx^{qm},$$

where $m$ is any positive integer assigned beforehand. The coefficients $c_2$, $c_3$, &c., are numerical, and it can be proved by the method of undetermined coefficients that

$$c_2=\tfrac{1}{2}p(p-q)/q^2, \ldots c_r=\left(\frac{p}{q}-r+1\right)c_{r-1}/r,$$

for $1<r<m+1$. By making $m$ an indefinitely large integer, $y$ becomes an infinite series, which is convergent for $|x|<1$. It remains to be proved that the sum of the infinite series $y$, when convergent, represents that branch of the function $(1+x)^{p/q}$ which reduces to 1 when $x$ is zero. This last part of the proof Dr. Boyd has failed to supply or even to indicate; the need of it will be seen when it is observed that when $y$ becomes an infinite series, the remainder R is also an infinite series, and it is essential to prove that, as $m$ increases indefinitely, the limit of R is zero.

It will not be amiss to observe that these criticisms, offered with all friendliness and sympathy, are provoked just *because* Dr. Boyd aims at a high standard of logical exactitude. Many a worse book than his may be said to have fewer faults—faults, that is, which lie on the surface and can be pointed out in a few words. To write a really sound book on algebra, not incomprehensible to the ordinary college student, and not hopelessly unscientific when judged from the standpoint of contemporary analysis, is a very difficult task. But it is a worthy one; and the attempt justifies itself, even if it is not crowned with unqualified success. The reader of Dr. Boyd's book cannot fail to gain many fruitful ideas; if he has mathematical capacity he will very likely apprehend them in a substantially correct form, even when the author's exposition is not entirely rigorous.

To sum up, we find in this treatise, as in others of its class, much that is fresh, vital and stimulating; an interest in the progress of research, and in the development of new conceptions; together with a style that is neither frivolous nor pedantic. What we miss is, on the one hand, the German thoroughness which spares no pains to make the logical chain of an argument complete, and, on the other, our English dexterity of manipulation. This last faculty is not of much importance, truly, but is worth reasonable cultivation. It is strange to us, for instance, to find a whole page spent on the decomposition of $x^4+px^2+q$ into a product $(x^2+\alpha)(x^2+\beta)$ without any reference to the fact that $x^4+px^2+q$ is a quadratic in $x^2$. It is only fair to say that, in this instance, the context partly accounts for the phenomenon; but other examples of needlessly complicated work could easily be given.

G. B. M.

## *A CANADIAN PIONEER IN SCIENCE AND EDUCATION.*

*Fifty Years of Work in Canada, Scientific and Educational.* By Sir William Dawson, C.M.G., LL.D., F.R.S. Pp. viii + 308. (London and Edinburgh: Ballantyne, 1901.)

LITTLE more than a year has passed since the friends of science and of education in Canada had to mourn the death of Sir William Dawson. Though for the last six years of his life he had retired from his active official duties, his pen was not allowed to remain idle, but continued to throw off papers for scientific journals, addresses to societies and books of a more or less popular kind. One of the occupations of these closing years appears to have been the preparation of a sketch of his own career, which he left complete even to the dated preface